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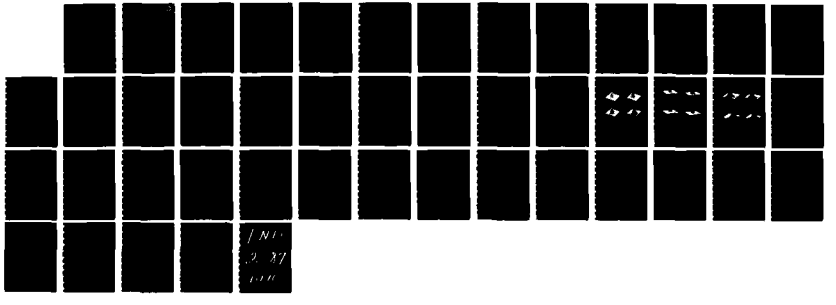

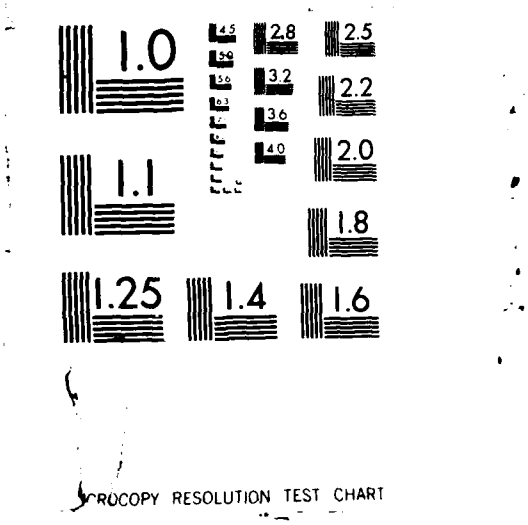
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Some Approximate Boltzmann Equation Solution Methods for the Study of Intense Microwave Pulse Propagation

P. J. PALMADESSO, M. J. MULBRANDON, J. CHEN, AND A. W. ALI

Plasma Physics Division

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CONTENTS

SOME APPROXIMATE BOLTZMANN EQUATION SOLUTION METHODS FOR THE STUDY OF INTENSE MICROWAVE PULSE PROPAGATION

I. INTRODUCTION

The propagation through the atmosphere of short pulse high power microwave (HPM) radiations is of interest for many applications. One of the important aspects of the propagation, however, is the attenuation of the pulse as it traverses the atmosphere. The attenuation results from the absorption of the radiation by the air plasma electrons, which are generated by the air breakdown action of the high power microwave. The microwave radiation is also attenuated through absorption by the air molecules and by scattering processes with molecules and particulates.

To calculate the attenuation of the HPM pulse one must solve Maxwell's equations in conjunction with the radiative transfer equation. The attenuation coefficient, due to the absorption by the air plasma, is related to the plasma conductivity or more specifically it depends on the electron density and the collision frequency for the momentum transfer. The electron density, on the other hand, depends on the avalanche ionization rate which in turn depends on the atomic cross sections and the appropriate electron velocity distributions.

The relevant quantities for the HPM attenuation calculations, however, can be obtained if an ab initio approach is considered, where one solves the Boltzmann equation in conjunction with Maxwell's equations and the radiative transfer model.

To obtain reasonable estimates for the microwave attenuation, one may use experimental values for the ionization and momentum transfer rates ν_i and ν_m. However, in the region of interest, i.e. high power and short pulse, there exist no experimental data. Therefore, the Boltzmann equation approach is necessary. The formulation of a Boltzmann equation requires a set of appropriate atomic cross sections which represent the state of the art and are reliable. The appropriate cross sections are given in tabular form at the end of this report based on a recent compilation by Ali[1].

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The Boltzmann equation approach in obtaining the electron velocity distribution has been formulated in general by many authors and its solution using spherical harmonics is well understood[2,3,4]. Many treatments of the Boltzmann equation, applied to breakdown in gases and swarm experiments, have utilized the steady state approach where ionization effects are not considered. This approach has also been used previously as means to obtain electron atom collision cross sections[5].

The effect of ionization for high E/N (E is the electric field and N the gas density) has been treated for swarm experiments by Taniguchi, et. al.,[6] Brunet and Vincent[7] and Yoshida, et. al[8]. The electron velocity distribution in gas breakdown by an rf pulse has been treated by Morez[9]. However, Boltzmann equation analyses for very high rf fields with short pulse durations have had very little attention. Recently, Pitchford[10,11] has considered the time dependent Boltzmann approach in the breakdown of nitrogen.

In the following sections, we describe some theoretical formulations which provide a basis for generating approximate solutions of the Boltzmann equation suitable for the study of short intense microwave pulses interacting with a background atmosphere, in various parameter regimes. It is not intended that all of these formulations be developed in full detail here; rather we shall simply outline calculation methods being considered in order to provide a platform for discussion of the advantages and disadvantages of the various approaches. One example, appropriate in the collisional regime, is developed in detail and implemented on a sample problem.

II. TIMESCALES

Microscopic and Macroscopic Timescales

We treat the problem by considering a short pulse of length L, where L/c is less than or of the order of a nanosecond. Before discussing the Boltzmann equation, it is useful to survey the various types of processes involved in the problem and the timescales on which each operates. There are several timescales relevant to the dynamics:

$\tau_\nu \sim 1/\nu$, where ν is an effective collision frequency; for the present we label all binary interactions as "collisions". ν may be less than, comparable to, or greater than ω.

$\tau_\omega \sim \lambda/c$. This is the wave period, the timescale associated with the interaction of a single electron with a cycle of the wave.

$\tau_p \sim L/c$. This is the transit time, the timescale associated with the interaction of a single electron with the pulse.

$\tau_w \sim \min(\tau_{w1}, \tau_{w2})$, where $\tau_{w1} = \lambda/|v_g - v_{ph}|$, $\tau_{w2} = \lambda/\Delta v_{ph}$, v_g and v_{ph} are the wave's group and central phase velocities, and Δv_{ph} is the width of the phase velocity spectrum of the wave packet. These are the timescales on which the waveform changes due to dispersion. The quantity $\lambda/|v_g - v_{ph}|$ is the time required for a point of constant phase to move a distance of order λ in the rest frame of the pulse, i.e. the frame moving with the group velocity. $\lambda/\Delta v_{ph}$ is the time required for dispersive effects to produce a local distortion in the shape of the waveform.

$\tau_d \sim L_d/c$, where L_d is the damping length. This is the wave damping or attenuation timescale.

$\tau_a \sim h/c$, where h is the atmospheric or ionospheric scale height. This is the timescale on which the ambient environment seen by the wavepacket changes.

Dispersion Timescale

For all cases of interest, $L \ll h, L_d$, so that $\tau_p \ll \tau_a, \tau_d$. For a narrowband wavepacket in a weakly dispersive medium, we also have $\tau_p \ll \tau_w$. To obtain a more quantitative feeling for the magnitude of the ratio τ_p/τ_w we estimate τ_w using an approximate dispersion relation. We note that for a wave in the 1 to 100 GHz range, and assuming an electron density of 10^8 electrons/cm^3 or less, we have

$$(\omega_p/\omega)^2 \ll 1,$$

where ω_p is the plasma frequency. In this case the dispersion relation is approximately

$$(kc/\omega)^2 = 1 - (\omega_p)^2/[\omega(\omega - i\nu)] + \text{terms of order } (\Omega/\omega)(\omega_p/\omega)^2, \qquad (1)$$

where Ω is the electron gyrofrequency, and ν is the effective collision frequency. Equation (1) can be used to show that

$$\tau_{w1} \sim \lambda(\omega^2+\nu^2)/[c(\omega_p\omega)^2].$$

Next we estimate τ_{w2}. The width of the k spectrum depends on the shape of the wave envelope, but $2\pi/L$ is a good estimate for practical cases. Then $\Delta v_{ph} = (\partial v_{ph}/\partial k)\Delta k$, and $\partial v_{ph}/\partial k$ can be calculated from equation (1). To lowest order in the small parameter $(\omega_p/\omega)^2$ one finds

$$\tau_{w2} \sim L(\omega^2+\nu^2)/[c(\omega_p\omega)^2].$$

Clearly $\tau_{w1} < \tau_{w2}$, since $\lambda < L$. Thus $\tau_w = \tau_{w1}$ and

$$\tau_p/\tau_w \sim (L/\lambda)[\omega_p\omega/(\omega^2+\nu^2)]^2. \qquad (2)$$

For a one nanosecond pulse at 1 GHz, $n_e \sim 10^8$/cc, $\nu = 0$, equation (2) yields $\tau_p/\tau_w \sim 10^{-2}$. We regard these as "worst case" parameters: higher frequencies, nonzero values of ν, and lower values of n_e all make this ratio smaller. In view of (2), we shall ignore the difference between the wave frame (v_{ph}) and the pulse frame (v_g) in the following.

III. QUASISTATIC BOLTZMANN EQUATION

Now consider the Boltzmann equation in a frame moving with the group velocity v_g. We assume that the direction of propagation is parallel to the z axis, and that spatial gradients in the x and y directions are negligible. In this reference frame we must use the relativistic Boltzmann equation, which is

$$\frac{\partial f}{\partial t} + vz\,\frac{\partial f}{\partial z} - e[E + (v/c)\times B]\cdot\frac{\partial f}{\partial p} = \left.\frac{\partial f}{\partial t}\right|_{coll.} \qquad (3)$$

In the wave frame the two dominant causes of temporal variation, wave-particle interactions and "collisions" (atomic processes) produce only spatial variations. In the absence of dispersion, attenuation, or inhomogeneities in the ambient environment, we would expect steady convective flow in the rest frame of the pulse, with ambient gas at all $z<0$ and a spatially growing disturbance between $z=0$ and $z=L$. We do not expect hydrodynamic turbulence to appear within the pulse, since the heavy particles cannot respond on a nanosecond timescale. Moreover, since v_z is close to c, and τ_w, τ_d, τ_a are all $\gg \tau_p$, it follows that

$$\partial/\partial t \sim 1/\min(\tau_w, \tau_d, \tau_a) \ll v_z\,\partial/\partial z \sim 1/\tau_p \text{ or larger.}$$

Thus the quasistatic Boltzmann equation,

$$v_z \frac{\partial f}{\partial z} - e(\mathbf{v}/c)\times\mathbf{B}\cdot\frac{\partial f}{\partial p} = \left.\frac{\partial f}{\partial t}\right|_{coll.} \tag{4}$$

in which the parameters of the pulse, the ambient medium, and the distribution function are all understood to be slowly varying functions of time, is an excellent approximation; the error incurred by dropping $\partial/\partial t$ in the pulse frame is of order 1% in the worst case, as shown above. Note that the electric field has been dropped from the equation, since $\mathbf{E} = -(1/c)\partial\mathbf{A}/\partial t$ vanishes in the wave frame; we recover $\mathbf{E}$ when transforming back to the lab frame. In the wave frame the total momentum, p, is a constant of the motion, since we have only a static magnetic field. Electrons and ions enter the pulse moving along the z axis at nearly the speed of light; the difference in their velocities is small. The magnetic field rotates the large wave frame electron velocity vector away from the ion velocity vector and toward the direction perpendicular to both z and the B-field direction (assume $\mathbf{B}$ is aligned with the y axis), and this appears in the lab frame as an acceleration in the x-direction. To an observer in the lab frame, it appears as if the magnetic force is weak, and that the acceleration is produced by a strong electric field in the x-direction

In the pulse frame the "information flow" is directed from the head of the pulse towards the tail, for all quantities. Thus, in addition to the reduction of the dimension of the system by one, we are now dealing with a one point boundary value problem, and we can seek solution algorithms which involve marching through the pulse from head to tail. In particular, it is possible to develop algorithms involving the solution of coupled sets of ordinary differential equations or quadratures. This is the essential attribute of the approach. In the following sections examples of such algorithms are outlined. In principle we can solve (4), use f to calculate the instantaneous current, electron density, conductivity, etc., as functions of the slowly varying ambient parameters, and use these in the Maxwell equations to self consistently determine the slowly varying pulse amplitude and waveform.

IV. REDUCTION TO SIMULTANEOUS ORDINARY DIFFERENTIAL EQUATIONS

The lab frame counterpart of equation (4) can be written in terms of the coordinate $\zeta = z - v_{ph}t$, where $v_{ph} = c$ is the phase velocity. We simply assume that the distribution function in the lab frame is a function of ζ and velocity components only; justification for this is provided by eq. (4). Then it follows that, in the lab frame,

$$(v_z - v_{ph}) \frac{\partial f}{\partial \zeta} - \frac{e}{m}[E(\zeta) + (v/c)\times B(\zeta)]\cdot\frac{\partial f}{\partial v} = \left.\frac{\partial f}{\partial t}\right|_{coll.} \qquad (4')$$

Coulomb or Fokker-Plank type collisions are unimportant on nanosecond timescales, for n_e values of interest here. The remaining "collision" effects can be expressed as follows:

$$\left.\frac{\partial f}{\partial t}\right|_{coll.} = \int f(\zeta,v')\, S(v,v')\, d^3v' - G(v)\, f(\zeta,v)$$

The first term on the right describes the rate at which particles are transported to the phase space location (ζ,v) from all other phase space locations (ζ,v'), by binary interaction processes. The second term gives the rate of loss from (ζ,v) due to such processes. The functions S and G, which describe atomic interactions, are assumed to depend on the slowly varying ambient parameters, but the associated functional dependences on z,t are suppressed here, in keeping with the quasistatic approximation.

Since f is square integrable, we can express it as a sum of orthogonal base functions in velocity space, with coefficients which vary with ζ. We assume that S and G can also be expanded.

$$f(\zeta,v) = \Sigma_{ijk}\, f_{ijk}(\zeta)\, \Phi_{ijk}(v) = \Sigma_J\, f_J(\zeta)\, \Phi_J(v)$$

$$G(v) = \Sigma_{ijk}\, G_{ijk}\, \Phi_{ijk}(v) = \Sigma_J\, G_J\, \Phi_J(v)$$

$$S(v,v') = \Sigma_{ijk}\Sigma_{i'j'k'}\, S_{ijki'j'k'}\, \Phi_{ijk}(v)\, \Phi_{i'j'k'}(v')$$

$$= \Sigma_J\Sigma_{J'}\, S_{J,J'}\, \Phi_J(v)\, \Phi_{J'}(v')$$

In general, the orthogonal functions for the expansion in a three dimensional velocity space would carry three indices (Φ_{ijk}), but since we truncate the series we can easily rearrange these into a one dimensional array with a single index (Φ_J), where the maximum value of J is N, the total number

of terms in the expansion. These relationships can be used to transform (4′) into a set of coupled ordinary differential equations for the coefficients $f_J(\zeta)$. In the simplest case, when $|v| \ll c$, we can neglect the magnetic field term in (4′) and set $(v_z - v_{ph}) \approx -c$ (these approximations simplify the analysis, but neither is absolutely necessary). Substituting the expansion relations above into (4′), performing the velocity integration, re-expanding remaining products of the form $\Phi_J(v)\Phi_{J'}(v')$ into sums of single Φ's, using recursion relations or re-expansion (depending on the choice of orthogonal functions) to express $\partial\Phi/\partial v$ in terms of Φ's, and equating coefficients of the orthogonal functions yields the desired set of N equations for the F's:

$$\frac{d}{d\zeta} fJ(\zeta) + TJ,J'(\zeta)\, fJ'(\zeta) = 0 \qquad (4'')$$

or,

$$\frac{d}{d\zeta} f(\zeta) + \underline{\underline{T}}(\zeta)\, f(\zeta) = 0$$

where $\underline{f}(\zeta)$ is the vector of expansion coefficients of $f(\zeta,v)$, and T is a matrix whose elements are functions of the electric field and the S and G expansion coefficients. In the case in which $\nu \gg \omega$, the distribution function will be nearly isotropic, in which case the number of separate equations to be solved and the size of the matrix T can be drastically reduced.

In any case, we solve by integrating from $\zeta = 0$ (ambient region ahead of the pulse) to $\zeta = L$. The accuracy of the resulting solution is limited by the amount of computation we are willing to do (manifested by the number of terms retained in expansions, other approximations employed, etc.), up to the accuracy limit of the quasistatic approximation itself (~1%), but it seems likely that good results can be achieved with a reasonable level of effort.

An alternative approach, in the same spirit, would be to represent the distribution function via a set of moments. One represents the distribution function by the product of a truncated series of orthogonal functions and a specified "approximation function" such as a Maxwellian or bi-Maxwellian with free parameters (drift velocity, temperature, etc.). A coupled set of ordinary differential equations for the free parameters and orthogonal function coefficients is obtained by the usual method. Convergence of the function series is enhanced when the "approximation function" is well chosen.

The coupled ordinary differential equation approach is likely to be the most practical method of generating solutions. It is possible, however, to

write down an analytic solution of the quasistatic equation in terms of a "scattering operator" Γ. This approach is discussed in the next section.

V. FORMAL SOLUTION OF THE QUASISTATIC EQUATION

We note that in the absence of collisions there are three constants of the motion for a particle in the force field of the pulse, in the wave frame. We have assumed the wave to be linearly polarized with B in the y-direction, so that in addition to the total energy the canonical momenta P_x and P_y are solutions of the homogeneous equation associated with (4). This means that we can simplify the operator on the left hand side of (4) by transforming to a new set of coordinates:

$$z,\ p_x,\ p_y,\ p_z \longrightarrow z,\ P_x,\ P_y,\ p$$

where

$$P_x = p_x + eA_x(z)/c$$

$$P_y = p_y$$

With the new coordinates, the quasistatic Boltzmann equation in the wave frame is

$$vz(z,C)\ \frac{\partial f}{\partial z}\bigg|_{\text{const. } C} = \frac{\partial f}{\partial t}\bigg|_{\text{coll.}} \tag{5}$$

where C is a shorthand representation of the set of constants P_x, P_y, p. The "collision" term can be expressed in the pulse frame as follows:

$$\frac{\partial f}{\partial t}\bigg|_{\text{coll.}} = \int f(z,C')\ S(z,C,C')\ d^3C' - G(z,C)\ f(z,C) \tag{6}$$

The first term on the right describes the rate at which particles are transported to the phase space location (z,C) from all other phase space locations (z,C'), by binary interaction processes. The second term gives the rate of loss from (z,C) due to such processes. Equations (5) and (6) can be combined and rewritten as follows:

$$\frac{df(z,C)}{dz} + g(z,C)\ f(z,C) = I(z,C) \tag{7}$$

where

$$I(z,C) = \int f(z,C')\ s(z,C,C')\ d^3C' \tag{8}$$

$$g(z,C) = [v_z(z,C)]^{-1} G(z,C) \tag{9}$$

$$s(z,C,C') = [v_z(z,C)]^{-1} S(z,C,C')$$

Note that $\partial/\partial z$ has been replaced by d/dz. Since the C's are to be held constant under the differentation by z, we can treat (7) as an ordinary first order differential equation in z, if we regard I(z,C) as a known function. In terms of I(z,C), the solution to (7) is

$$f(z,C) = e^{-\int g(z',C)dz'} \left[f(0,C) + \int_0^z I(z',C)\, e^{\int g(z'',C)dz''} dz' \right] \tag{10}$$

Now define the function space operator Γ such that

$$\Gamma f = \int_0^z \int f(z',C')\, s(z',C,C')\, d^3C'\, dz' \tag{11}$$

Eq. (11) becomes

$$e^{\int g(z',C)dz'} f(z,C) = f(0,C) + \Gamma e^{\int g(z',C)dz'} f \tag{12}$$

We can iterate (12) to obtain

$$e^{\int g(z',C)dz'} f(z,C) = [1 + \Gamma + \Gamma^2 + \Gamma^3 + \cdots]\, f(0,C) \tag{13}$$

We can formally "sum" (13) and express the solution to (12) in the form

$$f(z,C) = e^{-\int g(z',C)dz'} [1 - \Gamma]^{-1} f(0,C) \tag{14}$$

which can also be obtained directly from (12).

The operator Γ can be interpreted as a "scattering operator". In the absence of "collisions", the exact solution of the quasistatic equation is given by the first term of (13); the wavefield effects are carried in the constants of motion C(z,p). In a weak scattering situation, where electrons do not undergo more than one atomic interaction during their transit, Γ operating on the primary flux f(0,C) describes the contribution to f from electrons scattered into the interval (C,C+dC) after a single interaction, and we expect that $(1+\Gamma)f(0,C)$ will be an adequate solution. Similarly, we can interpret $\Gamma^2 f(0,C)$ as the contribution from electrons that enter (C,C+dC) after being scattered twice. $\Gamma^n f(0,C)$ is interpreted in analogous fashion.

The effects of losses from the (C,C+dC) interval are described in closed form by the $e^{-\int g dz}$ term. In general, if these interpretations are correct and if L_S is the mean free path for atomic processes which add particles to the phase space interval, then we expect that the series in (13) has converged after n terms if n is much larger than the expectation value for the number of scatterings experienced by an electron in moving from 0 to z, i.e. $n >> z/L_S$. To illustrate and test these ideas consider a simple model problem which can be solved exactly. Let f(z,C) be a function of z only, f = F(z), inside a region R of C-space with volume V_C, and zero outside this volume. Also take $g(z,C) = 1/L_G$ and $s(z,C,C') = 1/(L_S V_C)$. Equation (7) then has the solution

$$f(z,C) = F(z) = F(0) \exp[z/L_S - z/L_G] = F(0) \exp[z/L'] \quad \text{for C in R}$$

$$f(z,C) = 0 \quad \text{for C outside R}$$

where $$L' = L_S L_G/(L_G - L_S)$$

If $L_S < L_G$, then L' is positive. We have an avalanche if $0 < L' << L$.

Now let us consider (13). For the model problem, we find

$$\Gamma^n F(0) = (z/L_S)^n/n!\ F(0) \tag{15}$$

With (15), (13) is easily summed to yield the correct solution to the model problem. It follows from (15) that the ratio of the adjacent terms Γ^n/Γ^{n-1} in (13) is $(z/L_S)/n$, so that the convergence condition is the same as that inferred above. Note that the series always converges, even in the case of an avalanche.

We can express the scattering operator as a matrix by expanding in orthogonal functions, as before. We assume the orthogonality condition has the form (a slight modification of the analysis is needed if there is a weight function)

$$\int \Phi_J \Phi_{J'}\, d^3C = \delta_{J,J'}$$

Eq. (12) can be written in the form

$$Q(z,C) = f(0,C) + \Gamma Q(z,C) \tag{16}$$

where $$Q(z,C) = e^{\int g(z',C)dz'} f(z,C)$$

Differentiation of (16) with respect to z yields

$$\frac{dQ}{dz} = \int s(z,C,C')\, Q\, d^3C'$$

Now expand s and Q in orthogonal functions, and use the orthogonality relation to eliminate the integral. We get

$$d\underline{Q}/dz - \underline{\underline{s}}\,\underline{Q} = 0 \qquad (17)$$

where $$Q_J(z) = \int Q(z,C)\, \Phi_J(C)\, d^3C$$

$$s_{J,J'}(z) = \iint s(z,C,C')\, \Phi_J(C)\, \Phi_{J'}(C')\, d^3C\, d^3C'$$

The solution to (17) is

$$\underline{Q}(z) = \exp[\int \underline{\underline{s}}(z')dz']\,\underline{Q}(0) = \left(\sum_0^\infty [\int \underline{\underline{s}}(z')dz']^n/n! \right) \underline{Q}(0)$$

or, since $\underline{\Phi}\cdot\underline{Q} = \exp[\int g(z',C)dz']\, f(z,C)$ and $\underline{Q}(0) = \underline{f}(0)$,

$$f(z,C) = \underline{\Phi}\cdot\exp[\int \underline{\underline{s}}(z')dz' - \underline{\underline{I}}\int g(z',C)dz']\cdot\underline{f}(0) \qquad (18)$$

Eq. (18) is similar in form to the solution of the model problem considered above. For computation, one would replace the matrix part of the exponential in (18) by its series expansion [(18) is then the matrix version of (13)] and transform back to the lab frame. In its present form, (18) is more cumbersome for computational purposes than (4"), especially in the collisional regime, but it is possible that further analysis will lead to approximate algorithms useful in the weakly collisional regime where the series expansion converges rapidly.

In the next section we develop in some detail the equations needed to implement a version of the coupled ordinary differential equation approach discussed in section IV.

VI. COUPLED ORDINARY DIFFERENTIAL EQUATION CALCULATIONS

The General Formalism

In the laboratory frame, the Boltzmann equation can be written as

$$(v_z - v_{ph})\frac{\partial f}{\partial \zeta} - \frac{e}{m_e}\left[E(\zeta) + v/c \times B(\zeta)\right] \cdot \frac{\partial f}{\partial v} = \left.\frac{\partial f}{\partial t}\right|_{coll.} \tag{19}$$

where we have used m_e = electron mass, $\zeta \equiv z - v_{ph}t$ and $\partial f/\partial t = -v_{ph}(\partial f/\partial \zeta)$. In this section, we solve the above equation by reducing it to a system of ordinary differential equations. For the purpose of this report, which is to demonstrate the formalism, we will use a simplified scattering model. However, the basic technique is not limited to any particular scattering model.

Since $v_{ph} = c$ and $|v| << c$, we neglect the magnetic field term and set $(v_z - v_{ph}) = -c$ in the above equation. We also assume that the system is nonrelativistic. The coordinate system is such that z is the propagation direction of the microwave pulse and the electric field is in the x direction. Then, the approximate equation can be written as

$$\frac{\partial f}{\partial \zeta} + \frac{e}{m_e c} E_x(\zeta)\frac{\partial f}{\partial v_x} + \frac{1}{c}\left.\frac{\partial f}{\partial t}\right|_c = 0 \tag{20}$$

The last term describes the time rate of change of the distribution function due to collisions. In the present problem, we have partially-ionized plasmas so that the short-ranged collisions of electrons with neutral particles dominate. Coulomb collisions are unimportant. Thus, the collision term takes on the general form

$$\left.\frac{\partial f}{\partial t}\right|_{coll.} = \int d^3v' f(\zeta, v')S(v, v') - G(v)f(\zeta, v), \tag{21}$$

where the first term gives the time rate of increase in the distribution function $f(\zeta,v)$ due to collisions at all v' and the second term represents the time rate of loss of electrons at (ζ,v) due to collisions taking place at (ζ,v).

The plasma distribution function f is taken to depend on ζ and the spherical velocity coordinates v and θ, where θ is the polar angle. We define $w \equiv v_x/v = \cos\theta$ and use a separable form of f and the following set of basis functions:

$$\phi_j(v) \equiv \left(\frac{2}{v_o}\right)^{1/2} v^{-1} \sin\left(\frac{\pi j}{v_o} v\right), \quad j = 1,2,3,\ldots$$

and

$$P_m(w) \equiv \left(\frac{2m+1}{4\pi}\right)^{1/2} \hat{P}_m(w), \quad m = 0, 1, 2,3,\ldots$$

The normalization relations are

$$\int_0^{v_o} dv\, v^2 \phi_j(v)\phi_k(v) = \delta_{jk}$$

and

$$\int_{-1}^{1} dw P_m(w)P_n(w) = \frac{1}{2\pi}\,\delta_{mn}.$$

Here, v_o is the maximum electron velocity, and $\hat{P}_m(w)$ is the usual Legendre polynomials. We then exand f, S and G as follows:

$$f(\zeta,\underline{v}) = \sum_{i,l} f_{il}(\zeta)\phi_i(v)P_l(w)$$

$$S(\underline{v}, \underline{v}') = \sum_{i,l}\sum_{j,m} S_{iljm}\phi_i(v)\phi_j(v')P_l(w)P_m(w')$$

and

$$G(\underline{v}) = \sum_{i,l} G_{il}\phi_i(v)P_l(w).$$

Then, (20) can be written as

$$\sum_{i,l}\left(\frac{df_{il}}{d\zeta}\right)\phi_i(v)P_l(w) + \frac{eE_x(\zeta)}{m_e c}\sum_{i,l} f_{il}(\zeta)$$

$$\times \left[w\phi_i'(v)P_l(w) + \frac{1-w^2}{v}\,\phi_i(v)P_l'(w)\right]$$

$$+ \frac{1}{c}\left[\sum_{i,l}\sum_{j,m} S_{iljm} f_{jm}(\zeta)\phi_i(v)P_l(w)\right. \tag{22}$$

$$\left. - \sum_{i,l}\sum_{j,m} G_{il} f_{jm}(\zeta)\phi_i(v)\phi_j(v)P_l(w)P_m(w)\right] = 0,$$

where $\phi_i' = d\phi_i/dv$ and $P_l'(w) = dP_l/dw$. If we multiply all terms by the basis functions $\phi_j(v)P_m(w)$ and integrate over velocity space, then it is clear that a system of ordinary differential equations results.

A Simple Collision Model

In order to solve (22), we need to specify the collision terms. While this equation can be used for any scattering cross-sections, we will illustrate the method and its capabilities by using a simple collision model and solving the equation directly. In this paper, we assume that, after a collision with a neutral particle, an electron has an equal probability of being scattered into any solid angle. Then, we have

$$G(\underline{v}) = \bar{n}_a \sum_s \sigma_s(v), \tag{23}$$

and

$$S(\underline{v},\underline{v}') = \frac{\bar{n}_a}{4\pi}\, v^{-1} \sum_s \sigma_s(v')\ \delta\left(v' - \sqrt{v^2 + \varepsilon_s}\right), \tag{24}$$

where s is the index for various collision processes, $\bar{n}_a$ is the ambient neutral density and σ_s is the scattering cross-section (integrated over the solid angle) for the collision process s. The quantity ε_s is the normalized threshold energy for the interaction s; an electron with incoming energy of $(m_e v^2/2)$ loses $(m_e \varepsilon_s/2)$ as a result of the collision. For each interaction s, if the incident energy is less than ε_s, then no scattering takes place so that $\sigma_s(v) = 0$ for $v^2 < \varepsilon_s$. We define

$$\sigma_T(v) \equiv \sum_s \sigma_s(v) \tag{25}$$

Using (23) and (24), it is straightforward to calculate the expansion coefficients G_{il} and S_{iljm}. By multiplying all the terms in (22) by $\phi_j(v)P_m(w)$ and integrating over the velocity space, (22) can be reduced to the following system of differential equations,

$$\frac{df_{jm}}{d\zeta} + \left(\frac{2}{v_o}\right)\frac{eE_x(\zeta)}{m_e c}\left[-I^{(1)}_{m-1}\sum_i\left(\Delta_{ij} - \frac{1}{m}L_{ij}\right)f_{i,m-1}(\zeta)\right.$$

$$\left. + I^{(2)}_{m+1}\sum_i\left(\Delta_{ij} + \frac{1}{m+1}L_{ij}\right)f_{i,m+1}(\zeta)\right]$$

$$+ \chi_{jm}(\zeta) = 0, \tag{26}$$

where i,j,k = 1,2,3,.. and l,m,n = 0,1,2,3,.... The collision term χ_{jm} can be written as

$$\chi_{jm}(\zeta) \equiv \left(\frac{2}{v_o}\right)\left(\frac{\bar{n}_a}{c}\right)\delta_{mo}\sum_i f_{io}(\zeta)\bar{S}_{ji} - \left(\frac{2}{v_o}\right)^2\left(\frac{\bar{n}_a}{c}\right)\sum_i\sum_k \bar{G}_i f_{km}(\zeta)\bar{\Gamma}_{ikj} + \chi^{(i)}_{jm}(\zeta). \tag{27}$$

In the above expressions, the following definitions have been used:

$$\Delta_{ij} = \int_0^\pi dx\, x^{-1}\sin(ix)\sin(jx),$$

$$L_{ij} \underset{o}{\equiv} i\int_0^\pi dx\, \cos(ix)\sin(jx) = ij\left[1 - (-1)^{j+i}\right](j^2 - i^2)^{-1}$$

$$I_l^{(1)} \equiv \frac{(l+1)^2}{2l+1}\left(\frac{2l+1}{2l+3}\right)^{1/2},$$

$$I_l^{(2)} \equiv \frac{l^2}{2l+1}\left(\frac{2l+1}{2l-1}\right)^{1/2},$$

$$\bar{S}_{ji} \equiv \int_o^{v_o} dv \left[\sum_s \sqrt{v^2+\varepsilon_s}\,\sigma_s\left(\sqrt{v^2+\varepsilon_s}\right)\right]\sin\left(\frac{\pi j}{v_o}v\right)\sin\left(\frac{\pi i}{v_o}\sqrt{v^2+\varepsilon_s}\right)$$

$$\bar{G}_i \equiv \int_o^{v_o} dv\, v^2\sigma_T(v)\sin\left(\frac{\pi i}{v_o}\right)$$

and

$$\Gamma_{ikj} \equiv \int_o^{\pi} dx\, x^{-1}\sin(ix)\sin(kx)\sin(jx).$$

then we find

$$T = \sum_i \left[a_o \sum_j \frac{(-1)^{j+1}}{j}\bar{S}_{ji} - b_o\bar{G}_i\right]f_{io}(\zeta)$$

where

$$a_o = \frac{8\bar{n}_a}{c}\left(\frac{v_o}{2\pi}\right)^{1/2}$$

and

$$b_o = \frac{\bar{n}_a}{c}\left(\frac{8\pi}{v_o}\right)^{1/2}$$

using the definitions of $\bar{S}_{ji}$ and $\bar{G}_i$, it can be shown rigorously that

In (27), $\chi_{jm}^{(i)}(\zeta)$ describes the changes in the distribution function due to the creation of ionization secondary electrons. This term will be calculated later according to a simplified model. Note that, because of the isotropic scattering assumption, we have

$$S_{iljm} = \left(\frac{2}{v_o}\right) \bar{n}_a \delta_{lo} \delta_{mo} \bar{S}_{ij}$$

and

$$G_{il} = (4\pi)^{1/2} \left(\frac{2}{v_o}\right)^{1/2} \bar{n}_a \delta_{lo} \bar{G}_i$$

where δ denotes the δ-function. For more realistic scattering cross-sections of the form $\sigma_s(v,\theta)$, the summations in (26) would be more complicated. Note that conservation of particles requires that $\int d^3v (\partial f/\partial t)|_{coll.} = 0$ in the absence of ionization ($\chi_{jm}^{(i)} = 0$) for any distribution function f. If we define

$$T \equiv \int d^3v \, \frac{1}{c} \left.\frac{\partial f}{\partial t}\right|_{coll.}$$

$$\bar{G}_i = \frac{2v_o}{\pi} \sum_{j=1}^{\infty} \frac{(-1)^{j+1}}{j} \bar{S}_{ji} \tag{28}$$

for any choice of $\sigma_s(v)$, excluding ionization. Thus, $T = 0$ exactly for any f and any scattering processes.

Solution

We have solved the set of differential equations (26) by numerical integration. For the present work, we used 21 scattering processes to calculate $\bar{S}_{ji}$ and $\bar{G}_i$: they are ionization, vibrational excitations of N_2 molecules, electronic excitations of N_2 molecules, and momentum transfer. For the purpose of numerical computation, it can be shown that

$$\sum_i \bar{G}_i \Gamma_{ikj} = \left(\frac{v_o}{2}\right) \Lambda_{kj} \tag{29}$$

where

$$\Lambda_{kj} \equiv \int_o^{v_o} dv \, v\sigma_t(v) \sin\left(\frac{\pi k}{v_o} v\right) \sin\left(\frac{\pi j}{v_o} v\right).$$

Using (29), we can rewrite (27) as

$$\chi_{jm}(\zeta) = \left(\frac{2}{v_o}\right)\left(\frac{\bar{n}_a}{c}\right)\sum_i \left[\delta_{mo} f_{io}(\zeta)\bar{s}_{ji} - f_{im}(\zeta)\Lambda_{ji}\right] + \chi^{(i)}_{jm}(\zeta) \qquad (30)$$

This expression eliminates the double summation, providing better accuracy. For the purpose of illustrating the procedures involved in calculating the contribution of the secondary electrons created by ionization, we assume a simple model in which the secondary electrons are produced with zero energy. The result is expected to depend on the distribution of the secondary electrons, and for serious calculations one would use a more realistic distribution. Pitchford[10], for example, has assumed that the incident and secondary electrons share the energy equally following. In this paper, we will not discuss the detailed consequences of the model dependence; a task for future studies. If we assume that all secondary electrons are created with zero initial energy, then

$$\left.\frac{\partial f}{\partial t}\right|^{(i)}_{coll.} = \delta^3(v) \int G^{(i)}(v')\, f(\zeta, v')\, d^3v',$$

where the superscript (i) means that only the ionization cross-section is used. This expression gives the time rate of change of the distribution function due to the ionization secondary electrons. Using this expression, we find

$$\chi^{(i)}_{jm}(\zeta) = \left(\frac{2\bar{\pi}n_a}{cv^2_o}\right) j\delta_{mo} \sum_i \bar{G}_i^{(i)} f_{io}(\zeta), \qquad (31)$$

where $\bar{G}_i^{(i)}$ is obtained from the definition of $\bar{G}_i$ following (27) by replacing $\sigma_T(v)$ with $\sigma^{(i)}(v)$, the ionization cross-section.

Our code solves the set of equations represented by (26), with the collision terms $\chi_{jm}(\zeta)$ given by (30) and (31) or other expressions representing different assumptions about the nature of the "collisions". In particular, the assumption that all the secondary electrons are created with zero energy, discussed above as a simple illustration of the method, is numerically troublesome and somewhat unphysical: in practice the newly created electron population must have a finite temperature. The electron distribution

function can be reconstructed and displayed in phase space, or moments, conductivities, and other quantities of interest can be calculated directly from the function space representation.

For the purpose of evaluating the quantities in (26), we define equally spaced grids for the velocity and angular coordinates. Then, all quantities including the cross-sections, Δ_{ij}, $\bar{S}_{ji}$, and Λ_{ij} are defined and computed on these grids. For diagnostic purposes, we check particle and energy conservation.

Figures 1, 2, and 3 illustrate the type of results one obtains with this method, as applied to a sample problem: E_{wave} = 100 statvolts/cm., $\omega/2\pi$ = 30 GHz, and we have an atmosphere of N_2 with a density of $10^{18}/cm^3$. Secondary electrons are created with a thermal spread of 2 ev, in this test calculation. Figure 1 represents the drift of an initially Maxwellian distribution function with 20 ev temperature under the action of a wave electric field only (the wave amplitude has been reduced to 25 statvolts/cm. to avoid runaway): atomic processes are turned off. In Figure 2 we see the evolution of the same initial distribution function in the absence of a wave electric field, but with atomic processes operative. A tabulation of the cross section data used is provided in the Appendix. The "ledge" seen to form on the shoulders of the Maxwellian is due to vibrational excitations with cross sections sharply peaked near 2.5 ev. The growing central peak is primarily due to the cooling effect of the various excitation processes; i.e., these are mostly electrons scattered down from higher energies. Ionization of neutrals by hot tail particles also contributes to the growth of the central peak. In Figure 3, the wave (100 statvolts/cm. amplitude) and the atomic processes act simultaneously, and one can observe drift motions, the buildup of electron density due to ionization, etc. Note that the distribution function quickly becomes non-Maxwellian.

VII. SUMMARY AND COMMENTS

We have discussed an approach to the solution of the Boltzmann equation suitable for the study of short intense microwave pulses interacting with a background atmosphere in various parameter regimes, and a specific example appropriate in the collisional regime. The coupled ordinary differential equation approach appears to be viable. Continuing studies and comparisons of this approach with others are expected to expose the advantages and disadvantages of the method, and lead to refinements.

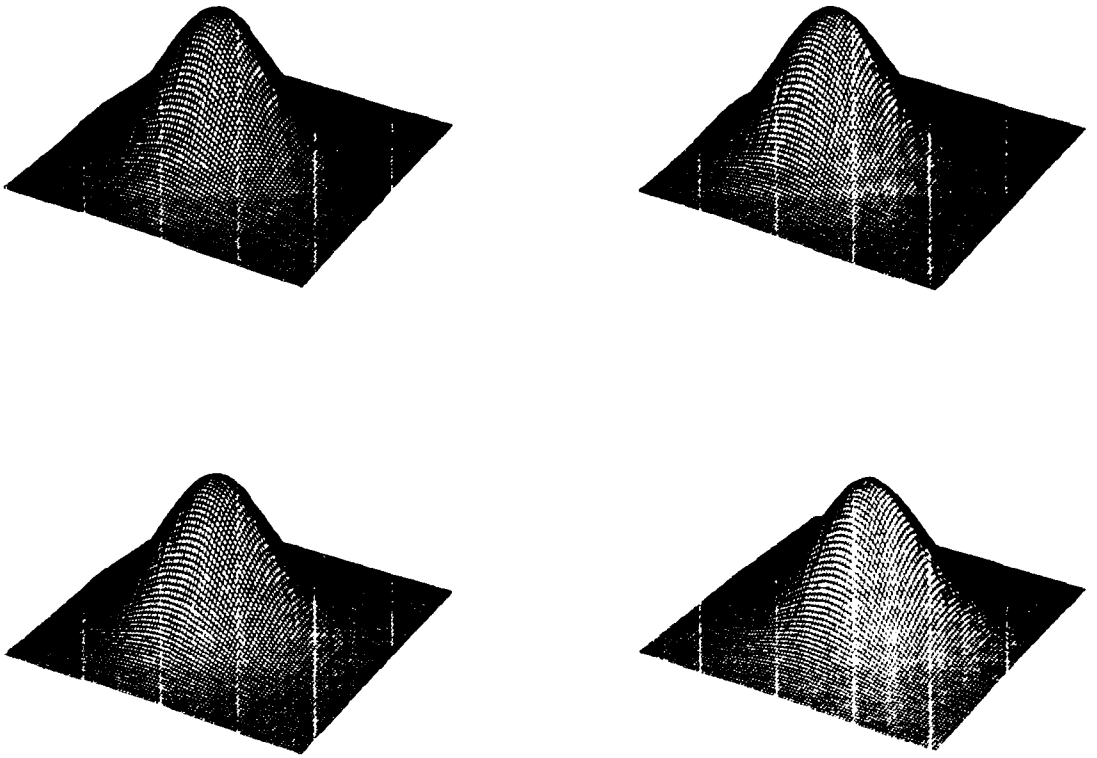

Figure 1. This Figure shows the drift of an initially Maxwellian distribution function with 20 ev temperature under the action of a wave electric field (E_{wave} = 25 statvolts/cm., $\omega/2\pi$ = 30 GHz) only: atomic processes are turned off. The four subfigures represent "snapshots" of the distribution at different wave phases, as the wave pulse moves past an element of the electron gas. The phases displayed are: (top left) $\zeta_1 = \lambda/4$, (top right) $\zeta_2 = \lambda/2$, (bottom left) $\zeta_3 = 3\lambda/4$, (bottom right) $\zeta_4 = \lambda$, where λ is the wavelength.

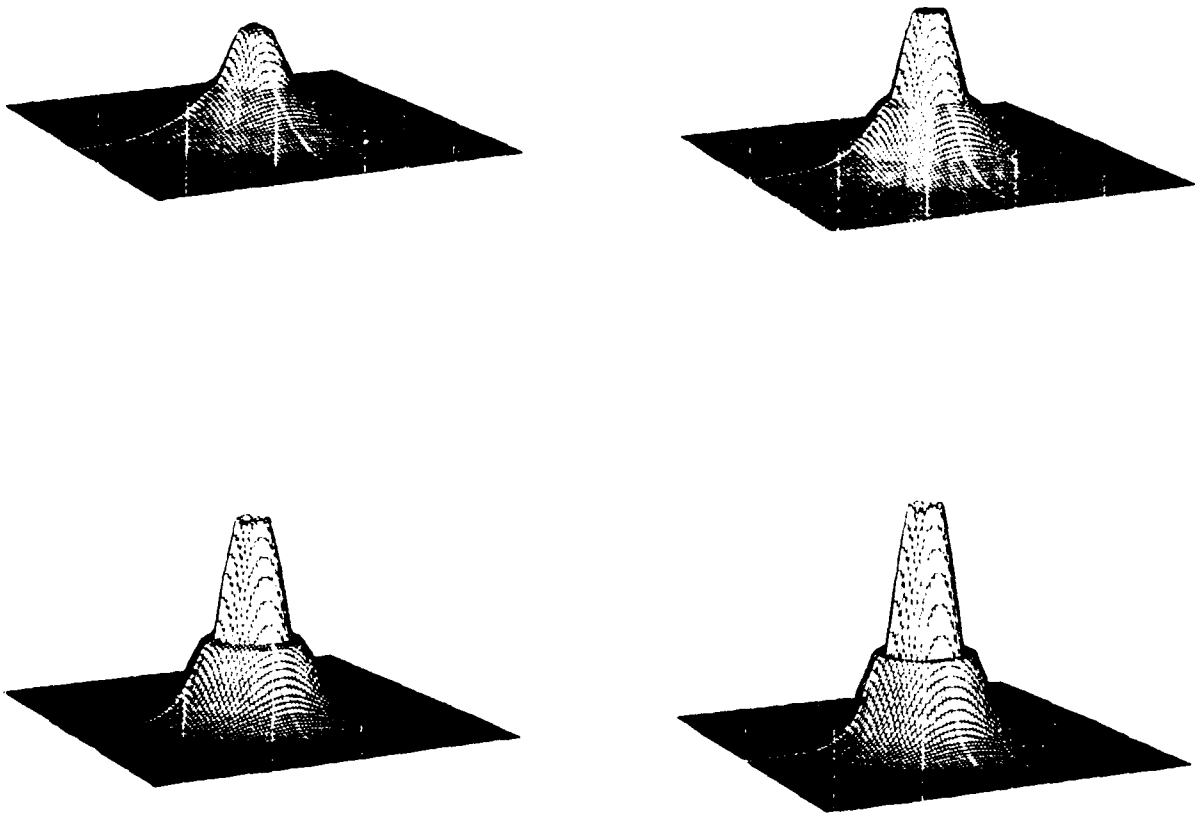

Figure 2. The evolution of a 20 ev Maxwellian distribution in the absence of a wave electric field, but with atomic processes operative is shown. The growing central peak is composed of primary electrons scattered down from higher energies and secondary electrons created when hot tail particles ionize neutrals. The "ledge" which forms on the shoulders of the Maxwellian is due to vibrational excitations with cross sections sharply peaked near 2.5 ev. Distributions are displayed at "times" ζ_i/c, i = 1,2,3,4.

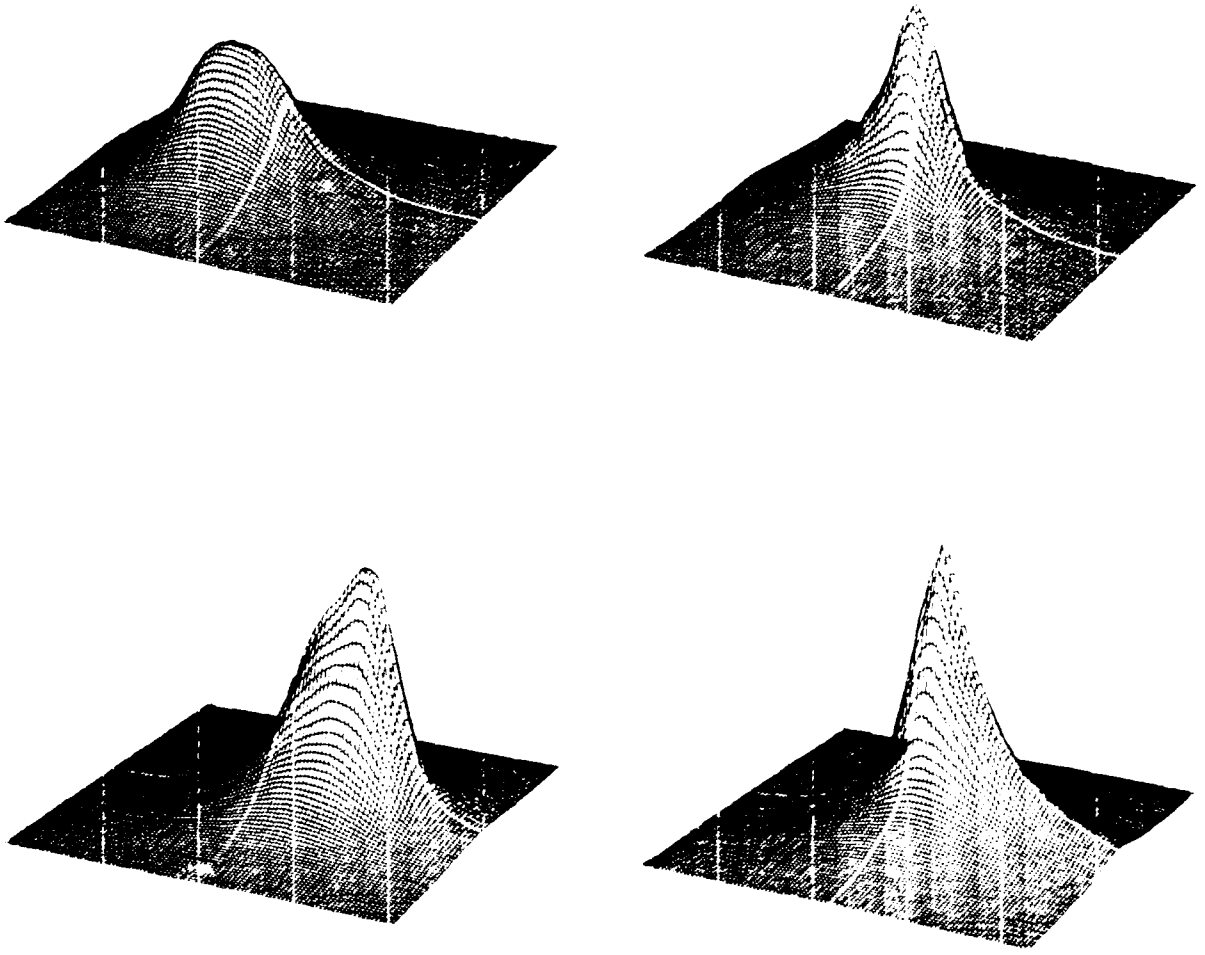

Figure 3. Evolution of the electron distribution function with a 100 statvolts/cm. wave and the atomic processes acting simultaneously. One can observe drift motions, the buildup of electron density due to ionization, etc. Note that the distribution function becomes non-Maxwellian. Again the subfigures represent snapshops at phases $\zeta = \lambda/4,\ \lambda/2,\ 3\lambda/4,\ \lambda$.

Acknowledgments

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APPENDIX: CROSS SECTION TABLES

Cross section data used in the sample calculation described in the text is tabulated below. The threshold energy for each process is given in parenthesis.

N_2 Rotational Cross Section

σ_0 (E_0 = .02 ev)

Electron Energy (eV)	$\sigma(cm^2)$
0.0	0.0
0.02	0.0
1.2	7.2(-18)
1.6	2.1(-17)
1.7	2.7(-17)
1.8	4.8(-17)
1.9	1.7(-16)
2.0	6.2(-16)
2.1	6.5(-16)
2.2	6.1(-16)
2.3	8.28(-16)
2.4	7.2(-16)
2.6	6.1(-16)
3.0	2.7(-16)
3.6	4.5(-17)
5.0	0

N_2 Vibrational Cross Section (v = 1)

σ_1 (E_1 = 0.29 ev)

Electron Energy (eV)	$\sigma(cm^2)$
0.3	1.32(-19)
0.75	6.4(-19)
1.44	1.22(-17)
1.65	1.84(-17)
1.74	2.30(-17)
1.85	4.3(-17)
1.92	1.01(-16)
1.98	2.60(-16)
2.02	3.0(-16)
2.09	2.1(-16)
2.12	1.46(-16)
2.18	7.8(-17)
2.24	1.38(-16)
2.27	2.36(-16)
2.31	2.8(-16)
2.38	2.18(-16)
2.41	1.39(-16)
2.47	1.07(-16)
2.54	1.2(-16)
2.57	1.78(-16)
2.61	2.0(-16)
2.66	1.66(-16)
2.71	3.6(-17)
2.73	8.8(-17)
2.76	5.6(-17)
2.79	7.0(-17)
2.83	3.68(-17)
3.07	7.68(-17)
3.27	3.98(-17)
3.60	3.38(-17)
4.0	0

N_2 Vibrational Cross Section (v = 2)

σ_2 (E_2 = 0.58 eV)

Electron Energy (eV)	$\sigma(cm^2)$
1.73	0.0
1.81	6.0(-18)
1.91	4.6(-17)
1.98	9.8(-17)
2.03	1.78(-16)
2.13	2.2(-16)
2.18	1.58(-16)
2.24	1.1(-16)
2.28	6.0(-17)
2.34	5.0(-17)
2.39	2.6(-17)
2.43	9.8(-17)
2.49	2.0(-16)
2.53	1.84(-16)
2.57	1.4(-16)
2.63	7.0(-17)
2.69	2.6(-17)
2.73	5.2(-17)
2.82	1.06(-16)
2.92	7.0(-17)
3.02	1.84(-17)
3.12	5.8(-17)
3.22	3.4(-17)
3.29	1.84(-17)
3.35	2.4(-17)
3.50	1.2(-17)
3.68	0.0

N_2 Vibrational Cross Section (v = 3)

σ_3 (E_3 = 0.87eV)

Electron Energy (eV)	$\sigma(cm^2)$
1.80	0.0
1.87	3.0(-18)
1.97	4.0(-17)
2.02	7.8(-17)
2.05	1.17(-16)
2.12	1.39(-16)
2.17	1.72(-16)
2.22	1.76(-16)
2.27	1.38(-16)
2.32	7.2(-17)
2.36	2.24(-17)
2.40	3.0(-18)
2.47	4.2(-17
2.52	6.9(-17)
2.57	8.7(-17)
2.6	9.6(-17)
2.67	6.9(-17)
2.72	2.7(-18)
2.77	3.0(-18)
2.82	3.0(-17)
2.92	5.1(-17)
3.02	2.1(-17)
3.13	1.5(-17)
3.21	1.94(-17)
3.32	3.0(-18)
3.35	0.0

N_2 Vibrational Cross Section (v = 4)

σ_4 (E_4 = 1.16 eV)

Electron Energy (eV)	$\sigma(cm^2)$
2.0	0.0
2.07	3.0(-18)
2.13	6.0(-17)
2.19	7.94(-17)
2.20	1.19(-16)
2.28	1.49(-16)
2.32	1.27(-16)
2.37	9.74(-17)
2.4	8.1(-17)
2.5	3.0(-18)
2.56	3.0(-18)
2.62	3.0(-17)
2.67	4.5(-17)
2.71	7.8(-17)
2.77	5.7(-17)
2.82	2.7(-17)
2.86	3.0(-18)
2.94	1.2(-17)
3.0	2.4(-17)
3.15	1.8(-17)
3.21	3.0(-18)
3.25	0.0

N_2 Vibrational Cross Section (v = 5)

σ_5 (E_5 = 1.45 eV)

Electron Energy (eV)	$\sigma(cm^2)$
2.0	0.0
2.13	3.0(-18)
2.17	3.9(-17)
2.22	5.7(-17)
2.32	8.1(-17)
2.38	9.3(-17)
2.42	9.9(-17)
2.48	9.9(-17)
2.52	8.1(1-17)
2.57	4.2(-17)
2.62	3.0(-18)
2.68	3.0(-18)
2.72	1.5(-17)
2.82	4.2(-17)
2.92	4.2(-17)
3.0	3.0(-18)
3.12	1.5(-17)
3.22	2.24(-17)
3.32	3.0(-18)
3.35	0.0

N_2 Vibrational Cross Section (v = 6)

σ_6 (E_6 = 1.74 eV)

Electron Energy (eV)	$\sigma(cm^2)$
2.25	0.0
2.30	3.0(-18)
2.37	4.2(-17)
2.40	7.0(-17)
2.44	9.0(-17)
2.49	9.06(-17)
2.54	9.06(-17)
2.65	8.44(-17)
2.72	4.22(-17)
2.81	1.81(-17)
2.9	1.5(-17)
3.0	2.7(-17)
3.1	1.2(-17)
3.15	0.0

N_2 Vibrational Cross Section (v = 7)

σ_7 (E_7 = 2.03 ev)

Electron Energy (eV)	$\sigma(cm^2)$
2.35	0.0
2.40	3.0(-18)
2.44	1.66(-17)
2.49	2.4(-17)
2.53	3.02(-17)
2.61	3.92(-17)
2.65	5.12(-17)
2.68	6.04(-17)
2.74	5.12(-17)
2.77	3.92(-17)
2.83	3.0(-17)
2.88	1.2(-17)
3.0	3.0(-18)
3.08	1.2(-17)
3.17	1.5(-17)
3.3	9.0(-18)
3.4	0.0

N_2 Vibrational Cross Section (v = 8)

σ_8 (E_8 = 2.32 eV)

Electron Energy (eV)	$\sigma(cm^2)$
2.50	0.0
2.57	3.0(-18)
2.62	1.8(-17)
2.66	2.42(-17)
2.71	2.42(-17)
2.78	1.96(-17)
2.84	2.4(-17)
3.0	9.0(-18)
3.04	3.0(-18)
3.15	3.0(-18)
3.3	9.0(-18)
3.4	3.0(-18)
3.5	0.0

Excitation Cross section For $N_2(A^3\Sigma)$

σ_9 (E_9 = 6.17 eV)

Electron Energy (eV)	σ (cm^2)
6.17	0.0
6.50	1.28(-18)
7.0	1.88(-18)
8.0	6.3(-18)
9.0	1.21(-17)
10	1.68(-17)
11	2.22(-17)
12	1.92(-17)
13	1.5(-17)
14	1.2(-17)
15	1.0(-17)
16	8.8(-18)
18	6.7(-18)
20	5.4(-18)
25	3.8(-18)
30	2.5(-18)
35	1.7(-18)
40	1.3(-18)

Above 40 eV use

$$\sigma = \frac{8.06 \times 10^{-14}}{E^3}$$

Excitation Cross section For $N_2(B^3\Pi)$

σ_{10} (E_{10} = 7.35 eV)

Electron Energy (eV)	$\sigma(cm^2)$
7.35	0.0
7.5	4.0(-19)
8.0	5.4(-18)
9.0	1.4(-17)
10.0	2.25(-17)
11.0	2.78(-17)
12.0	2.99(-17)
13.0	2.97(-17)
14	2.71(-17)
15	2.41(-17)
16	2.16(-17)
17	1.95(-17)
18	1.79(-17)
19	1.66(-17)
20	1.56(-17)
22	1.42(-17)
24	1.30(-17)
26	1.20(-17)
28	1.10(-17)
30	1.01(-17)
32	9.2(-18)
34	8.4(-18)
40	5.1(-18)

Above 40 eV use

$$\sigma = 3.3 \times 10^{-13} \frac{1}{E^3}$$

Excitation Cross section For $N_2(w^3\Delta)$

σ_{11} (E_{11} = 7.36 eV)

Electron Energy (eV)	$\sigma(cm^2)$
7.36	0.0
8	2.7(-18)
9	7.4(-18)
10	1.2(-17
11	1.66(-17)
12	2.13(-17)
13	2.6(-17)
14	3.06(-17)
15	3.51(-17)
16	3.8(-17)
17	3.76(-17)
18	3.5(-17)
19	3.09(-17)
20	2.65(-17)
22	1.97(-17)
24	1.53(-17)
26	1.26(-17)
28	1.08(-17)
32	8.35(-18)
36	6.6(-18)
38	5.9(-18)
40	5.2(-18)
50	3.0(-18)

Above 50 eV use

$$\sigma = 3.75 \times 10^{-13} \frac{1}{E^3}$$

Excitation Cross section For $N_2(B'^3\Sigma)$

σ_{12} (E_{12} = 8.16 eV)

Electron Energy (eV)	$\sigma(cm^2)$
8.16	0.0
9	1.6(-18)
10	3.5(-18)
11	5.5(-18)
12	7.4(-18)
13	9.4(-18)
14	1.13(-17)
15	1.25(-17)
16	1.14(-17)
17	9.2(-18)
18	7.3(-18)
19	6.1(-18)
20	5.4(-18)
22	4.7(-18)
24	4.3(-18)
26	3.9(-18)
30	3.4(-18)
34	2.9(-18)
40	2.4(-18)
44	2.2(-18)
50	1.9(-18)

Above 50 eV use

$$\sigma = 2.37 \times 10^{-13} \frac{1}{E^3}$$

Excitation Cross section For $N_2(a'^1\Sigma)$

σ_{13} (E_{13} = 8.4 eV)

Electron Energy (eV)	$\sigma(cm^2)$
8.39	0.0
9.0	1.0(-18)
10	2.7(-18)
11	4.5(-18)
12	6.2(-18)
13	8.0(-18)
14	9.6(-18)
15	1.04(-17)
16	8.5(-18)
17	6.4(-18)
18	5.2(-18)
19	4.5(-18)
20	4.1(-18)
22	3.4(-18)
26	2.7(-18)
30	2.3(-18)
34	2.0(-18)
38	1.87(-18)
42	1.84(-18)
46	1.82(-18)
50	1.80(-18)

Beyond 50 eV use

$\sigma = 9 \times 10^{17} \left(\frac{1}{E}\right)$

Excitation Cross section For $N_2(a^1\Pi)$

$\sigma_{14}(E_{14} = 8.55 \text{ eV})$

Electron Energy (eV)	$\sigma(cm^2)$
8.54	0.0
9	9.5(-19)
10	3.0(-18)
11	4.95(-18)
12	7.0(-18)
13	9.0(-18)
14	1.1(-17)
15	1.3(-17)
16	1.43(-17)
17	1.50(-17)
18	1.49(-17)
19	1.43(-17)
20	1.38(-17)
22	1.26(-17)
26	1.14(-17)
30	1.02(-17)
34	9.2(-18)
38	8.8(-18)
42	7.6(-18)
46	6.9(-18)
50	6.3(-18)

Beyond 50 eV use

$$\sigma = \frac{3 \times 10^{16}}{E}$$

Excitation Cross section For $N_2(w^1\Delta)$

σ_{15} (E_{15} = 8.87 eV)

Electron Energy (eV)	$\sigma(cm^2)$
8.89	0.0
9	2.0(-19)
10	3.7(-18)
11	7.1(-18)
12	9.9(-18)
13	1.77(-17)
14	1.15(-17)
15	1.0(-17)
16	8.1(-18)
17	6.6(-18)
18	5.6(-18)
20	4.3(-18)
24	3.2(-18)
28	2.6(-18)
32	2.1(-18)
36	1.6(-18)
40	1.3(-18)
44	1.0(-18)
48	8.0(-19)
50	7.0(-19)

Beyond 50 eV use

$$\sigma = \frac{1.87 \times 10^{-15}}{E^2}$$

N_2 Dissociation Cross Section

σ_{16} (E_{16} = 10.0 eV)

Electron Energy (eV)	$\sigma(cm^2)$
9.7	0.0
10.0	1.27(-18)
10.5	1.87(-18)
11	2.49(-18)
12	4.92(-18)
13	1.12(-17)
14	2.43(-17)
15	3.58(-17)
16	4.75(-17)
18	6.82(-17)
20	8.56(-17)
22	1.05(-16)
26	1.36(-16)
30	1.58(-16)
35	1.67(-16)
40	1.73(-16)
50	1.75(-16)
60	1.79(-16)
70	1.80(-16)
90	1.71(-16)
100	1.64(-16)

After 100 eV use

$$\sigma = \frac{1.64 \times 10^{-14}}{E^2}$$

Excitation Cross section For $N_2(C^3\Pi)$

σ_{17} (E_{17} = 11.03 ev)

Electron Energy (eV)	$\sigma(cm^2)$
11.03	0.0
11.5	8.0(-18)
12	1.46(-17)
13	2.98(-17)
14	4.43(-17)
15	3.89(-17)
16	2.84(-17)
17	2.34(-17)
18	2.02(-17)
19	1.81(-17)
20	1.65(-17)
22	1.39(-17)
24	1.18(-17)
28	8.6(-18)
32	6.6(-18)
36	5.2(-18)
40	4.2(-18)
44	3.4(-18)
48	2.8(-18)
50	2.6(-18)

Beyond 50 eV use

$$\sigma = \frac{1.42 \times 10^{-14}}{E^{2.2}}$$

Excitation Cross section For $N_2(E^3\Sigma)$

σ_{18} (E_{18} = 11.87 ev)

Electron Energy (eV)	$\sigma(cm^2)$
11.87	0.0
12	5.0(-20)
13	1.0(-19)
14	2.1(-19)
15	3.0(-19)
16	4.0(-19)
17	5.0(-19)
18	5.6(-19)
19	6.2(-19)
20	7.0(-19)
22	7.8(-19)
24	8.0(-19)
28	6.5(-19)
32	4.0(-19)
36	2.7(-19)
40	1.8(-19)
44	1.0(-19)
48	8.0(-20)
50	7.0(-20)

Beyond 50 eV use
$\sigma = 0$

Ionization Cross Section

σ_{19} (E_{19} = 15.6 ev)

Electron Energy (eV)	$\sigma(cm^2)$
16	2.1(-18)
16.5	4.65(-18)
17	7.12(-18)
17.5	9.84(-18)
18	1.29(-17)
18.5	1.63(-17)
19	1.98(-17)
19.5	2.3(-17)
20	3.07(-17)
20.5	3.07(-17)
21	3.45(-17)
22	4.17(-17)
23	4.91(-17)
24	5.65(-17)
25	6.38(-17)
26	7.13(-17)
28	8.74(-17)
30	1.03(-16)
32	1.15(-16)
36	1.38(-16)
40	1.57(-16)
45	1.77(-16)
50	1.93(-16)
60	2.18(-16)
70	2.33(-16)
85	2.46(-16)
100	2.52(-16)
120	2.52(-16)
160	2.42(-16)
180	2.35(-16)
200	2.27(-16)
300	1.92(-16)
400	1.66(-16)
500	1.45(-16)
600	1.29(-16)
800	1.06(-16)
1000	9.22(-17)

Beyond 1000 eV use

$$\sigma = \frac{2.09 \times 10^{-14}}{E} \left[\log (0.08E) - \log (1 - \beta)^2 - \beta^2\right]$$

$$\beta = \frac{v}{c}$$

Momentum Transfer Cross Section

σ_m

Electron Energy (eV)	$\sigma_m(cm^2)$
0.01	2.2(-16)
0.014	2.5(-16)
0.022	2.94(-16)
0.032	3.5(-16)
0.065	4.9(-16)
0.1	6.04(-16)
0.23	8.2(-16)
0.33	9.3(-16)
1.0	9.98(-16)
1.5	1.15(-15)
1.6	1.25(-15)
1.8	1.60(-15)
2.0	1.8(-15)
2.2	1.8(-15)
2.6	1.5(-15)
2.8	1.4(-15)
3.0	1.25(-15)
3.3	1.10(-15)
3.6	1.05(-15)
4.0	9.8(-16)
4.5	9.2(-16)
5.0	9.1(-16)
6.0	1.0(-15)
7.0	1.05(-15)
9.0	1.0(-15)
10	9.4(-16)
15	8.4(-16)
20	8.2(-16)
30	6.0(-16)
50	4.1(-16)
100	2.0(-16)
200	9.0(-17)
400	3.5(-17)
700	1.25(-17)

Beyond 700 eV use

$$\sigma_m = \frac{8.75 \times 10^{-15}}{E}$$

END

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